Exercise 7.2 Prove the following sequents and theorems, using the primitive rules of predicate logic. You may also use derived sentential rules.

S116 a=b ⊢ b=a

S117 a=b & b=c ⊢ a=c

*S118 a=b, b≠c ⊢ a≠c

S119 Fa & ∀x(Fx → x=a), ∃x(Fx & Gx) ⊢ Ga

*S120 ∀x x=x → ∃xFx, ∀x(∼Fx ∨ Gx) ⊢ ∃x(Fx & Gx)

S121 ∀x(Fx → Gx), ∀x(Gx → Hx), Fa & ∼Hb ⊢ a≠b

*S122 ∃x((Fx & ∀y(Fy → y=x)) & Gx), ∼Ga ⊢ ∼Fa

S123 ∃x∀y((∼Fxy → x=y) & Gx) ⊢ ∀x(∼Gx → ∃y(y≠x & Fyx))

S124 ∃x(Px & (∀y(Py → y=x) & Qx)), ∃x∼(∼Px ∨ ∼Fx) ⊢ ∃x(Fx & Qx)

*S125 ∀x∃yGyx, ∀xy(Gxy → ∼Gyx) ⊢ ∼∃y∀x(x≠y → Gyx)

T60 ⊢ ∀x ∃y x=y

T61 ⊢ ∀x(Fx ↔ ∃y(x=y & Fy))

T62 ⊢ ∀x(Fx ↔ ∀y(x=y → Fy))

T63 ⊢ ∀xy(Rxy ↔ x=y) → ∀xRxx