*S63 $(P \to Q) \to R, S \to ({\sim}Q \to T) \vdash R \lor {\sim}T \to (S \to R)$

*S64 $(P \to Q) \,\&\, (R \to P), (P \lor R) \,\&\, {\sim}(Q \,\&\, R) \vdash (P \,\&\, Q) \,\&\, {\sim}R$

S65 $P \,\&\, Q \to (R \lor S) \,\&\, {\sim}(R \,\&\, S), R \,\&\, Q \to S,$
$S \to ((R \,\&\, Q) \lor ({\sim}R \,\&\, {\sim}Q)) \lor {\sim}P \vdash P \to {\sim}Q$

S66 ${\sim}(P \,\&\, {\sim}Q) \lor {\sim}({\sim}R \,\&\, {\sim}S),$
${\sim}S \,\&\, {\sim}Q, T \to ({\sim}S \to {\sim}R \,\&\, P) \vdash {\sim}T$

*S67 $P \,\&\, Q \to R \lor S \vdash (P \to R) \lor (Q \to S)$

*S68 $P \,\&\, Q \to (R \lor S) \,\&\, {\sim}(R \,\&\, S), R \,\&\, Q \to S,$
$S \to ((R \,\&\, Q) \lor ({\sim}R \,\&\, {\sim}Q)) \lor {\sim}P \vdash P \to {\sim}Q$

S69 $(P \to Q) \to (Q \to P) \vdash (P \to Q) \to ({\sim}P \to {\sim}Q)$

2.3 Theorems

Theorem

Definition. A **THEOREM** is a sentence that can be proved from the empty set of premises.

Comment. We can assert that a given sentence is a theorem by presenting it as the conclusion of a sequent with nothing to the left of the turnstile.

Example.

Prove $\vdash P \,\&\, Q \to Q \,\&\, P$.

1	(1)	P & Q	A
1	(2)	Q	1 &E
1	(3)	P	1 &E
1	(4)	Q & P	2,3 &I
	(5)	$P \,\&\, Q \to Q \,\&\, P$	$4 \to$I (1)

Comment. Note that in step 5 we discharge assumption 1. Hence, the final conclusion rests on no assumptions (i.e., the assumption-set is the empty set).