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Comment. Requiring that the sequent to be used as a derived rule be proved using only primitive rules is unnecessarily restrictive. If the sequents are proved in a strict order and no later sequent in the series is used in the proof of an earlier sequent, then no logical errors can result. We suggest the stronger restriction only because it is good practice to construct proofs using only the primitive rules.

Comment. Since the ten primitive rules are truth-preserving, it follows that any derived rule is also truth-preserving.

Exercise 2.2.3 Prove the following using either primitive or derived rules from the previous exercises. If you like a challenge, prove them again using primitive rules only.

S45 $\sim P \rightarrow P \dashv\vdash P$

S46 $P \leftrightarrow Q \dashv\vdash \sim((P \rightarrow Q) \rightarrow \sim(Q \rightarrow P))$

*S47 $P \leftrightarrow Q \dashv\vdash P \vee Q \rightarrow P \,\&\, Q$

*S48 $P \leftrightarrow Q \dashv\vdash \sim(P \vee Q) \vee \sim(\sim P \vee \sim Q)$

S49 $P \leftrightarrow Q \dashv\vdash \sim(P \,\&\, Q) \rightarrow \sim(P \vee Q)$

S50 $P \leftrightarrow Q \dashv\vdash \sim(\sim(P \,\&\, Q) \,\&\, \sim(\sim P \,\&\, \sim Q))$

S51 $P \vee Q \rightarrow R \,\&\, \sim P,\ Q \vee R,\ \sim R \vdash C$

S52 $\sim P \leftrightarrow Q,\ P \rightarrow R,\ \sim R \vdash \sim Q \leftrightarrow R$

S53 $\sim((P \leftrightarrow \sim Q) \leftrightarrow R),\ S \rightarrow P \,\&\, (Q \,\&\, T),\ R \vee (P \,\&\, S)$
$\vdash S \,\&\, K \rightarrow R \&\, Q$

*S54 $(P \,\&\, Q) \vee (R \vee S) \vdash ((P \,\&\, Q) \vee R) \vee S$

S55 $P \,\&\, (\sim Q \,\&\, \sim R),\ P \rightarrow (\sim S \rightarrow T),\ \sim S \rightarrow (T \leftrightarrow R \vee Q) \vdash S$

S56 $P \,\&\, \sim Q \rightarrow \sim R,\ (\sim S \rightarrow \sim P) \leftrightarrow \sim R \vdash R \leftrightarrow Q \,\&\, (P \,\&\, \sim S)$

*S57 $P \vee Q,\ (Q \rightarrow R) \,\&\, (\sim P \vee S),\ Q \,\&\, R \rightarrow T \vdash T \vee S$

*S58 $P \rightarrow Q \vee R,\ (\sim Q \,\&\, S) \vee (T \rightarrow \sim P),\ \sim(\sim R \rightarrow \sim P) \vdash \sim T \,\&\, Q$

S59 $P \vee Q,\ P \rightarrow (R \rightarrow \sim S),\ (\sim R \leftrightarrow T) \rightarrow \sim P \vdash S \,\&\, T \rightarrow Q$

*S60 $(P \leftrightarrow \sim Q) \rightarrow \sim R,\ (\sim P \,\&\, S) \vee (Q \,\&\, T),\ S \vee T \rightarrow R \vdash Q \rightarrow P$

*S61 $\sim S \vee (S \,\&\, R),\ (S \rightarrow R) \rightarrow P \vdash P$

*S62 $P \vee (R \vee Q),\ (R \rightarrow S) \,\&\, (Q \rightarrow T),\ S \vee T \rightarrow P \vee Q,\ \sim P \vdash Q$